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Preventing Environmental Disasters: Market-Based vs. Command-and-Control Policies

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Preventing Environmental Disasters:

Market-Based vs. Command-and-Control Policies

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Abstract

The paper compares the effects of market-based and command-and-control climate policies on the direction of technical change and the prevention of environmental disasters. Drawing on the model proposed in Acemoglu et al. (2012, American Economic Review), we show that market-based policies (carbon taxes and subsidies towards clean sectors) exhibit bounded window of opportunities: delays in their implementation make them completely ineffective both in redirecting technical change and in avoiding environmental catastrophes. On the contrary, we find that command-and-control interventions guarantee policy effectiveness irrespectively on the timing of their introduction. As command-and-control policies are always able to direct technical change toward "green" technologies and to prevent climate disasters, they constitute a valuable alternative to market-based interventions.

JEL: O33, O44, Q30, Q54, Q56, Q58

Keywords: Environmental Policy, Command and Control, Carbon Taxes, Disasters

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1 Introduction

In this work, we extend the seminal contribution of Acemoglu et al. (2012)[1] to study the effectiveness of market-based (M-B) and command-and-control (C&C) policies in redirecting technical change towards "green" innovations, thus preventing environmental catastrophes related to climate change.

One of the major challenges faced by humankind today is the rising temperature caused by the increasing consumption of fossil fuels, and the appropriate policy responses. The debate is still unsettled as some researchers call for major and immediate actions (Stern, 2007), whereas others suggest limited and gradual policy interventions (see e.g. Nordhaus, 2007).

Acemoglu et al. (2012) contribute to such debate with a two-sector model of directed technical change, which allows to study how market-based environmental policies can affect the development of "dirty" and "green" technologies, thus impacting on climate change. When the clean and dirty inputs are "strong " substitute (more on that in Acemoglu et al., 2012, and in Section 4.3 below.), an optimal M-B environmental policy, grounded on a "carbon" tax and a green research subsidy, can redirect technical change towards the green sector, preventing environmental catastrophes. However, given path-dependency in the direction of technical change (Aghion et al., 2015), the window of opportunity of such policy actions is limited: if the technology gap between the dirty and green inputs becomes sufficiently high, M-B interventions are ineffective and environmental disasters will certainly occur. The policy window is shorter when the two inputs are "weak" substitutes. Moreover, in the latter case (as well as when inputs are complementary) M-B interventions cannot avoid an environmental catastrophe unless they stop the growth of the economy.

We expand the model of Acemoglu et al. (2012) to account for command-and control policies and we study their impact on technical change and climate dynamics. The adoption of regulations not grounded on market incentives is quite common in environmental policy and it appears to be very effective. For instance, some international agreements (e.g. the Montreal Protocol on Substances that Deplete the Ozone Layer) fix an exogenous ceiling on specific polluting concentrations. Shapiro and Walker (2015) find that the increasing stringency of U.S. environmental regulation accounts for

[1] For an extension see also Acemoglu et al. (2015).

three quarters of the 60% decrease in pollution emissions (e.g. nitrogen oxides, particulate matter, sulfur dioxide, and volatile organic compounds) from U.S. manufacturing in the period from 1990 to 2008. More generally, both price and quantity forms of control can solve allocation problems and there is no a priori criterion to favor one instrument over the other (Weitzman, 1974). The choice of the most appropriate mode of control should be grounded only on its performance evaluated according to "economic" criteria.

We show that a command-and-control policy, which fixes the maximum amount of dirty inputs for each unit of clean ones, is *always* able to redirect technical change towards the green technology independently on the timing of its implementation, and to avoid environmental catastrophes. In that, C&C policies are a valuable alternative to M-B ones to reach a green transition whenever the window of opportunity for environmental interventions is bounded. Moreover, even if the dirty and clean inputs are weak substitutes or complementary, command-and-control interventions imposing a ceiling to the use of polluting inputs, avoid disasters without halting economic growth.

The rest of the paper is organized as follows. We present a literature review in Section 2 and the model in Section 3. Alternative policies interventions are compared in Section 4. Finally, we discuss the results in Section 5.

2 Literature review

The optimal policy to induce a large-scale transition from dirty to clean production system is — in the absence of any other market failure — a Pigouvian price on the polluting substances, typically a carbon tax on emissions (Nordhaus, 1991). Moreover, in a world with perfect knowledge and no uncertainty, the standard duality argument implies the equivalence between the use of prices and quantities as mitigation policy instruments. However, Weitzman (1974) shows that in an imperfect world, the relative advantage of price policies vis-à-vis quantity ones can vary according to the amount and type of inadequate information and uncertainty. As a consequence, quantity-based mitigation policies could be more efficient that price-based ones.

Relatedly, many studies have compared the effects of market-based (M-B) and command-and-control (C&C) policies (see Hepburn, 2006, for an extensive comparison of environmental and climate policies). Buchanan (1969); Li and Shi (2010) and Li and Sun (2015) emphasize the drawbacks of marked-based instruments and support the use of regulation. The same conclusion is suggested by recent empirical evidences (see e.g. Lee, Veloso and Hounshell, 2011; Shapiro and Walker, 2015). On the contrary, Rozenberg, Vogt-Schilb and Hallegatte (2014) show that when underinvestment in polluting capital is possible, carbon pricing is superior to C&C interventions. More generally, there is no crystal-clear agreement about the identification of the "best" climate policy and the way to compare different instruments. For example, Goulder and Parry (2008) consider a wide range of possible interventions, and they analyze the extent they meet a variety of major evaluation criteria, including cost-effectiveness, distributional equity, the ability to address uncertainties, and political feasibility. They find that no single instrument is clearly superior and that ensembles of different climate policies can be more effective.

In a seminal contribution, Acemoglu et al. (2012) study how directed technical change can support the transition to a "green" economy and prevent environmental disasters (see Popp, 2004, for an earlier contribution). They find that the joint adoption of carbon taxes and research subsidies can direct innovation towards clean technologies (on the complementarity of tax and subsidies see also Grimaud, Lafforgue and Magn, 2011).

The fundamental role of innovation for the effectiveness of climate and environmental policy has been explored extensively both at theoretical (Goulder and Schneider, 1999; Gillingham, Newell and Pizer, 2008; Otto and Reilly, 2008) and empirical levels (Popp, 2002; Jaffe, Newell and Stavins, 2003).[2] Such contributions point out that the rewards from the development of new technologies do come not from current or future pollution reductions, but from the expected profits associated with technological improvements. Expected profitability depends in turn on the size of the economy (Sue Wing, 2003), the scarcity of fossil fuels (Gans, 2012), and the past history of innovations (Acemoglu et al., 2012; Aghion et al., 2015).

In particular, Acemoglu et al. (2015) show that if the initial technological gap between dirty and clean technologies is too wide, the potential transition to clean technology cannot occur as clean research must climb several steps to catch up with dirty technology and the gap discourages

[2]For an excellent review on technological change and the environment see also Popp, Newell and Jaffe (2010).

research efforts in "green" technology. As a consequence, the path-dependency of technical change should be considered in assessing the effectiveness of different climate policy instruments to induce "green" transitions, as well as to prevent environmental disasters related to excessive greenhouse gasses emissions. This is exactly the starting point of the present paper. In addition, whenever history plays a relevant role, the timing of policy interventions is crucial. While Gerlagh, Kverndokk and Rosendahl (2009) show that the introduction of optimal emission reduction policies strongly depends on the set of instruments available, we take the opposite perspective and study how policies can loose their effects as time goes by.

3 The model

The baseline structure of the model is akin to the one in Acemoglu et al. (2012). There is a continuum of households (composed of workers, entrepreneurs and scientists) with utility function:

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t), \tag{1}$$

where ρ is the discount rate, C is final consumption good, and $S \in [0, \bar{S}]$ captures the quality of the environment. Naturally, the instantaneous utility function is increasing in consumption and environmental quality.[3]

On the supply side, a homogeneous final good is produced under perfect competition employing *clean* and *dirty* inputs Y_c and Y_d:

$$Y_t = \left(Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where $\epsilon \in (0, +\infty)$ is the elasticity of substitution between the two inputs. Note that the two inputs are complements when $\epsilon < 1$ and substitutes if $\epsilon > 1$.

Both Y_c and Y_d are produced using labor and a continuum of sector-specific machines according to the production functions:

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di \tag{3}$$

[3]The utility function is twice differentiable and jointly concave in C and S. Moreover, conditions (2) and (3) in Acemoglu et al. (2012) hold. Households' utility considerable falls when environmental quality approaches zero. Conversely, if $S = \bar{S}$, further increases of environmental quality do not lead to utility improvements.

with $j \in \{c, d\}$ and $\alpha \in (0, 1)$, A_{jit} is the productivity of machine i in sector j and x_{jit} is the quantity of such machine. The aggregate productivity of the two sectors is defined as:

$$A_{jt} \equiv \int_0^1 A_{jit} di. \tag{4}$$

Total labor supply is normalized to 1 and the market clearing condition for labor requires $L_{ct}+L_{dt} \leq 1$. Machines in both clean and dirty sectors are produced by monopolistic competitive firms. The cost of producing a single machine is constant across time and sectors and corresponds to $\psi = \alpha^2$.

In both industry, an innovation occurs if a scientist successfully discovers a new design. At the beginning of each period scientists try to develop a new clean or dirty technology. If she is successful, which happens with probability $\eta_j \in (0, 1)$, she obtains a one-year patent for its machine i and becomes a monopolistic supplier.[4] Innovations increase the productivity of a machine by a factor $1 + \gamma$, with $\gamma > 0$. Normalizing the number of scientists to 1 the market clearing condition for scientists becomes: $s_{ct} + s_{dt} \leq 1$, where s_{jt} indicates the share of scientists conducting research in sector $j = \{c, d\}$ at time t. As scientists are randomly allocated to machines in the sector they choose, the average productivity of sector j evolves according to

$$A_{jt} = (1 + \gamma\eta_j s_{jt})A_{jt-1}. \tag{5}$$

The variation of environmental quality S_t depends on environmental degradation linked to the production of dirty inputs, as well as on environmental regeneration due to the intrinsic dynamics of the Earth's physical and biological system:

$$S_{t+1} = -\xi Y_{dt} + (1 + \delta)S_t, \tag{6}$$

with S_{t+1} bounded between 0 and $\overline{S}$. The environmental degradation term catches the negative effects of CO_2 emissions,[5] while the environmental regeneration term captures the absorption of CO_2 by the oceans and the biosphere (Oeschger et al., 1975; Goudriaan and Ketner, 1984; Nordhaus, 1992). Note that if $S_t = 0$ an environmental disaster occurs.

[4] In sectors where the innovation process is unsuccessful, a one-year patent is randomly assigned to one of the producers using the old technology.

[5] One could reasonably define CO_2 emissions as directly proportional to the use of dirty inputs: $Em_t \propto Y_{dt}$.

4 Climate policies and the direction of technical change

In this section we first recall the *laissez-faire equilibrium* (Section 4.1), where no environmental policies are in place. We then study the impact of different environmental policies aiming at redirecting technical change towards the green sector in order to reduce the total amount of dirty inputs used in the economy and avoid environmental disasters. More specifically, we compare the success of market-based policies (cf. Section 4.2), based on carbon tax and subsidies to the clean sector, vis-à-vis command-and-control interventions (cf. Section 4.3), which fix ceilings for the production of dirty inputs.

4.1 The laissez-faire equilibrium

As in Acemoglu et al. (2012), an equilibrium is represented by a sequence of wages (w_t), prices for inputs (p_{jt}) and machines (p_{jit}), demands for inputs (Y_{jt}) and machines (x_{jit}), labor (L_{jt}), quality of environment (S_t) and research allocations of scientists (s_{jt}) such that: firms maximize their profits, scientists maximize their expected profits, labor and input markets clear, and environmental quality evolves according to (6). We recall that the laissez-faire equilibrium occurs when no environmental policies are in place.

In line with Acemoglu et al. (2012), let us assume that the productivity of the green sector is sufficiently lower than the one of the dirty industry:

Assumption 1. $\frac{A_{c0}}{A_{d0}} < \min\left((1+\gamma\eta_c)^{-\frac{\varphi+1}{\varphi}}\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}, (1+\gamma\eta_d)^{\frac{\varphi+1}{\varphi}}\left(\frac{\eta_c}{\eta_d}\right)^{\frac{1}{\varphi}}\right),$

with $\phi \equiv (1-\alpha)(1-\epsilon)$. Assumption 1 will hold throughout the rest of the paper. If $\epsilon > 1$, innovation occurs only in the dirty sector and the long run growth rate of dirty inputs is $\gamma\eta_d$. If $\epsilon < 1$, innovation happens first in the clean sector, then it will occur also in the fossil fuel one and the long run growth rate of dirty inputs will be $(\eta_d\eta_c)/(\eta_d+\eta_c) < \eta_d$. If assumption 1 holds and $\epsilon > 1$, the laissez-faire allocation will always produce an environmental disaster, i.e. $S_t = 0$ for some t (Acemoglu et al., 2012).

Note that the direction of technical change is determined by the incentives scientists face when they decide to conduct their research in the clean or dirty sector. More specifically, the relative benefit from undertaking research in sector c rather than in sector d is expressed by the ratio (Acemoglu et al., 2012):

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \left(\frac{p_{ct}}{p_{ct}}\right)^{\frac{1}{1-\alpha}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct-1}}{A_{dt-1}}. \tag{7}$$

Equation (7) reveals that the relative profitability of research in the two sectors can be decomposed in three components which capture productivity differentials (A_{ct-1}/A_{dt-1}), relative prices ($(p_{ct}/p_{dt})^{\frac{1}{1-\alpha}}$), and market size ($L_{ct}/L_{dt}$). Finally, the equilibrium demand of the two inputs is determined by:

$$Y_c = (A_c^{\varphi} + A_d^{\varphi})^{-\frac{\alpha+\varphi}{\varphi}} A_c^{\alpha+\varphi} A_d, \tag{8}$$

$$Y_d = (A_c^{\varphi} + A_d^{\varphi})^{-\frac{\alpha+\varphi}{\varphi}} A_d^{\alpha+\varphi} A_c, \tag{9}$$

whose evolution over time depends on the sectoral allocation of scientists (cf. Equation 7), and on the stochastic process characterizing the dynamics of machines' productivity.

4.2 Market-based environmental policies

In the model, a market-based (M-B) environmental policy is composed of a carbon tax and a subsidy towards clean research proportional to firms' profits. When the two inputs Y_c and Y_d are substitutes ($\epsilon > 1$) and the economy is initially stacked in the *bad* laissez-faire equilibrium, Acemoglu et al. (2012) shows that the social planner can redirect technical change towards the green technology introducing a carbon-tax t_d on the production of dirty inputs and a public subsidy q_{ct} supporting the research in the clean sector.[6] Moreover, if the two inputs are strong substitutes ($\epsilon > 1/(1-\alpha)$), M-B policy is temporary.

However, the direction of technical change is not only dependent on policy variables (t_{dt} and q_{ct}). In fact, the past history of innovations, which in turns determines the relative productivity of the clean and dirty sectors and the profitability of performing research therein, plays a fundamental role. As a consequence, given the importance of path dependency (Aghion et al., 2015), even if

[6]Throughout the model we assume that subsidies have to be finite and the carbon tax, expressed as a percentage of the price of dirty inputs, must be lower than one. In particular $t_{dt} \leq \theta < p_{dt}$ and $q_{ct} \leq \vartheta < +\infty$. Notice that if such assumption does not hold, the model collapses to a degenerate case of a one-sector economy where the issue of directed technical change becomes completely meaningless.

carbon taxes and subsidies might affect the direction of technical change, they cannot always push the economy away from a bad, carbon-intensive equilibrium.

Proposition 1. *Assume $\epsilon > 1$ and that Assumption 1 holds. Let (q_{ct}, t_{dt}) be a policy scheme composed by a finite subsidy q_{ct} for the clean sector and a carbon tax t_{dt} on the production of dirty inputs lower than their unitary price, $t_{dt} < p_{dt}$. Then, there exists a finite $\bar{A}_d > 0$ s.t. $\forall A_{dt} > \bar{A}_d$*

(i) (q_{ct}, t_{dt}) is ineffective in re-directing technical change towards the clean sector,

(ii) the unique equilibrium allocation of scientists is $s_{dt} = 1$ and $s_{ct} = 0$ for any $t > 0$.

Proof. See Appendix A.1. □

Proposition 1 shows that market-based environmental policies fail whenever the productivity differentials between the dirty and clean sectors is sufficiently large. The intuition behind such a result is that once the relative productivity advantage of dirty technology is sufficiently big, it will always compensate the cost of the carbon tax and the benefit of the subsidy. This happens because the policy scheme affects the relative profitability of clean technology only via the market size effect. Accordingly, potential entrepreneurs will always maximize expected profits by investing in the carbon-intensive sector, thereby undermining the effectiveness of the carbon tax. Furthermore, the next proposition states that the probability of passing the productivity threshold $\bar{A}_d$, above which market-based policies are ineffective, approaches 1 as time goes by

Proposition 2. *Assume that $\epsilon > 1$ and that Assumption 1 holds. Then*

$$\lim_{t \to +\infty} P(A_{dt} > \bar{A}_d \,|\, t \in \boldsymbol{N}) = 1$$

Proof. See Appendix A.2 □

The main consequence of Proposition 2 is that the timing of the introduction of market-based environmental policies is crucial. If the economy is stacked in the bad equilibrium ($s_d = 1$), then there exists a bounded window opportunity for policy scheme (q_{ct}, t_{dt}) to be effective. More precisely, the window opportunity for a market based policy introduced at time T lasts $(\log(\frac{\overline{A}_d}{A_{dT}}))/(\log(1+\eta_d\gamma))$ periods.

Remark 1. Given the evolution of the quality of the environment (Equation 6), a too much delayed policy intervention inevitably leads to an environmental disaster. Indeed, as the economy is stacked in a bad equilibrium ($s_d = 1$), Y_{dt} increases at a rate of $\gamma\eta_d$, M-B policies become ineffective and S_t reaches zero in finite time, thereby producing a disaster.

Remark 2. The higher the elasticity of substitution between the two inputs (ϵ), the shorter the time window in which M-B policies are effective.

The intuition underlying the last remark is straightforward. If the elasticity of substitution of clean and dirty inputs increases, producers will have higher incentives to switch towards the cheaper dirty ones, creating additional demand for these inputs. Accordingly, the relative profitability of dirty inputs increases, reducing the threshold $\overline{A}_d$ above which policy interventions are ineffective.

Carbon taxes are a meaningful tool for emission control policy. At the same time, Proposition 1 and 2 show that, when markets are competitive and dirty and clean inputs are substitutes, their effectiveness cannot be guaranteed a priori. The productivity gap between carbon-intensive and green technologies becomes crucial for the success of M-B environmental policies. In addition such a gap widens over time due to the sub-martingale nature of machines' productivity (cf. proof of Proposition 2), implying that delayed interventions are likely to be ineffective.

Table 1 provides numerical experiments supporting these claims. It considers different scenarios defined by the expected productivity growth, initial relative backwardness of clean technologies and the size of the subsidy. In many cases, the window of opportunity for market-based policy lasts very few periods and, the carbon tax needed to redirect technical change is extremely high. For example, when the productivity of clean technologies is initially half of the dirty ones and the expected productivity growth is high, the carbon tax required to move the economy towards the "green" equilibrium is above 35% and it has to be introduced by 30 periods. Would the backwardness of clean technologies be higher, the minimum tax would correspond to 61% of dirty sector's revenues and the window of opportunity reduces to around 20 periods.

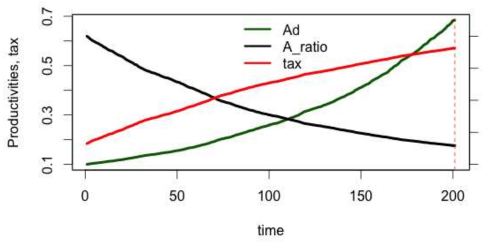
A further insight on the potential shortness of M-B's effectiveness period is provided by figure 1. Numerical simulations of the model under different technological regimes (corresponding to the

Table 1: Minimum carbon taxes to redirect technical change and corresponding window of opportunities (in parenthesis) under different policy schemes.

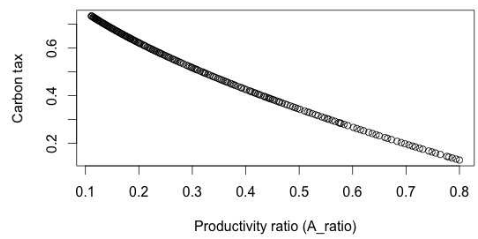
Backwardness of clean technologies	*Expected productivity growth*	*Subsidy (proportion of clean sector' profits)* 20%	10%	5%
low (70% of dirty)		19.0% (118)	20.4% (117)	20.0% (117)
medium (50% of dirty)	*low*	33.8% (106)	34.4% (105)	34.7% (105)
high (20% of dirty)		61.8% (76)	62.1% (75)	62.3% (75)
low (70% of dirty)		21.6% (36)	22.3% (36)	22.6% (36)
medium (50% of dirty)	*high*	35.9% (33)	36.5% (32)	36.8% (32)
high (20% of dirty)		63.0% (23)	63.3% (22)	63.5% (22)
low (70% of dirty)		30.5% (33)	31.1% (33)	31.4% (33)
medium (50% of dirty)	*asymmetric*	43.2% (30)	43.7% (30)	43.9% (30)
high (20% of dirty)		67.2% (20)	67.5% (20)	67.6% (20)

Low, medium and high expected productivity growth correspond to an average growth rate of machines' productivity of, 1%, 3% and 8% respectively. With asymmetric expected productivity growth, they corresponds to 8% for dirty technologies and 3% for clean ones.

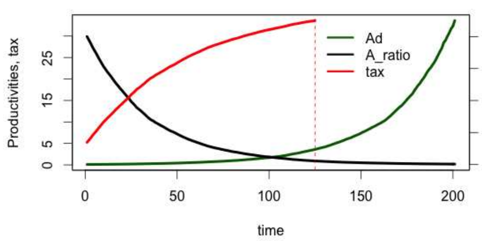
Figure 1: Window opportnities, minimum carbon tax and productivities under different technological regimes



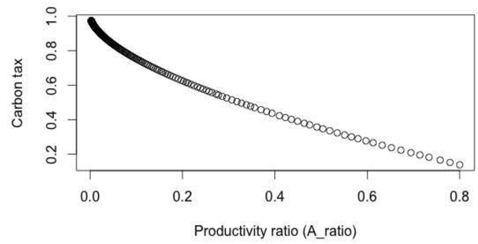
(a) $\eta_d = 0.1$, $\gamma = 0.1$



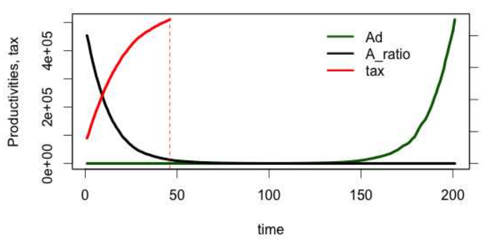
(b) $\eta_d = 0.1$, $\gamma = 0.1$



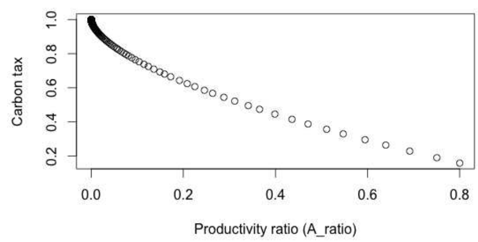
(c) $\eta_d = 0.3$, $\gamma = 0.1$



(d) $\eta_d = 0.3$, $\gamma = 0.1$



(e) $\eta_d = 0.4$, $\gamma = 0.2$



(f) $\eta_d = 0.4$, $\gamma = 0.2$

figures on the left column (a,c,e) show the Monte Carlo average dynamics of productivities and minimum carbon tax to redirect technical change obtained across 100 independent runs; figures on the right column (b,d,f) show the relationship between average carbon taxes and the ratio between clean and dirty sector productivity, each point represents a period. Each row of figures is characterized by different technological regimes: low (a,b) corresponds to an average productivity growth of 1%, medium (c,d) corresponds to an average productivity growth of 3%, high (e,f) corresponds to an average productivity growth of 8%. All simulations are obtained setting $\epsilon = 10$, $\alpha = 1/3$ and initial the initial productivity ratio $A_{c0}/A_{d0} = 0.8$.

technological opportunities in the two sectors) allow to illustrate the dynamics of productivities. The latter in turns determine the magnitude of the minimum carbon tax necessary to put the economy on a "green" development path. Simulation results show that, in many circumstances, the minimum tax rate rapidly reaches the unit, becoming ineffective as a policy instrument.

Given such dismal results, are there alternative policy interventions that always redirect technical change towards the green sector, thus preventing climate catastrophes? In the next Section, we will show that such objectives can be achieved by appropriate command-and-control policies.

4.3 Command-and-control policies

A command-and-control (C&C) policy refers to an environmental intervention that relies on regulation (permission, prohibition, standard setting and enforcement) as opposed to financial incentives, that is, economic instruments of cost internalization (UN, 1997). In particular, we consider a policy that establishes the maximum amount κ of dirty inputs that can be used for each unit of clean ones:[7]

$$\frac{Y_{ct}}{Y_{dt}} > \kappa, \quad \forall t > T, \tag{10}$$

where $\kappa \in \mathbf{R}^{+}$ represents the command-and-control policy chosen by the government and introduced at time T. Given Assumption 1, in the laissez-faire equilibrium innovations start in the more productive dirty sector. Moreover, if $\epsilon > 1$ and given Equations (8) and (9), C&C policies (cf. Equation 10) will be binding in equilibrium. The dirty (Y_{dt}) and clean (Y_{ct}) inputs are employed competitively by final good producers, which maximizes their profits according to:

$$\max_{Y_{ct},Y_{dt}} \; \{p_t Y_t - p_{ct} Y_{ct} - p_{dt} Y_{dt}\} \quad s.t. \;\; Y_t = \left(Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{11}$$

Under the C&C policy, first-order conditions give:

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{p_c}{p_d}\right)^{-\epsilon} = \kappa. \tag{12}$$

In both the laissez-faire equilibrium and market-based policies, the relative demand of inputs is determined competitively (the relative price of clean inputs compared to dirty ones is decreasing

[7] The effects of such policy on the direction of technical change are equivalent to the ones of an absolute upper bound for the use of dirty inputs. We refer to the relative threshold per unit of clean inputs to simplify computations.

in its relative supply). In contrast, under command-and-control policies, the government indicates a relative upper bound on the use of dirty inputs (κ), which implicitly constrains the gap between the prices of the two sectors. Furthermore, relying on (12), it is possible to express the relative employment in the clean sector as

$$\frac{L_{ct}}{L_{dt}} = \frac{Y_{ct}}{Y_{dt}} \left(\frac{p_{ct}}{p_{dt}}\right)^{-\frac{\alpha}{1-\alpha}} \left(\frac{A_{ct}}{A_{dt}}\right)^{-1} = \kappa^{(1+\frac{\alpha}{\epsilon-\alpha\epsilon})} \left(\frac{A_{ct}}{A_{dt}}\right)^{-1}. \tag{13}$$

Equation (13) implies that whenever the relative demand of dirty inputs is constrained by the C&C policy, any productivity gain in the carbon intensive sector is labor destroying. Indeed, since firms cannot expand production, any increases in machine productivity will increase profits by reducing the number of employees needed to serve a constant demand. The profitability ratio of conducting research in the two sectors than becomes:

$$\begin{aligned} \frac{\Pi_{ct}}{\Pi_{dt}} &= \frac{\eta_c}{\eta_d} \left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct-1}}{A_{dt-1}} \\ &= \frac{\eta_c}{\eta_d} \kappa^{\frac{\epsilon-1}{\epsilon}} \left(\frac{A_{ct}}{A_{dt}}\right)^{-1} \frac{A_{ct-1}}{A_{dt-1}} = \frac{\eta_c}{\eta_d} \kappa^{\frac{\epsilon-1}{\epsilon}} \left(\frac{1+\eta_c s_{ct}}{1+\eta_d s_{dt}}\right)^{-1}, \end{aligned} \tag{14}$$

where the second equality follows combining (7) with (12) and (13), and the third one is obtained via (5). From the last equation it follows that under a command-and-control policy κ the expected profitability of the two sectors in equilibrium does not depend on the productivity of currently available machines as in the laissez-faire and market-based policy cases, but only on the relative likelihood of obtaining a successful innovation and κ. We can now state the following proposition.

Proposition 3. *Consider a C&C policy κ. If $\epsilon > 1$, then there exists a finite $\bar{\kappa} > 0$ such that any $\kappa > \bar{\kappa}$ always redirects technical change towards the green sector.*

Proof. See Appendix A.3 □

The main intuition behind Proposition 3 is that any C&C policy, which sufficiently limits the relative share of dirty inputs, creates an additional demand for clean ones, increasing the profitability of the green sector. If such a policy is implemented, technical change moves towards a development path where innovations occur only in the clean sector. Moreover, once the new equilibrium is achieved, the economy behaves as in the "good" laissez faire scenario, where output Y_t and use of clean inputs Y_{ct} grows at the long-run rate $\gamma\eta_c$ (see equations 8, 9 and 4 and recall that A_{ct} grows at a rate of $\gamma\eta_c$ while A_{dt} remains constant).

Remark 3. A *temporary* C&C policy intervention κ is sufficient to redirect technical change permanently.

Indeed, as the economy moves toward a "good" equilibrium, Y_{ct} grows faster than Y_d, as the clean sector is more profitable than the dirty one, thus increasing the ratio Y_{ct}/Y_{dt}. The command-and-control policy sustains the improvement of machines' productivity in the clean sector, thereby increasing the ratio A_{ct}/A_{dt}. When such ratio becomes sufficiently high, the C&C policy is not needed anymore as research is spontaneously performed only in the clean sector.

Command-and-control policies can always redirect technical change towards the clean sector, but are they able to prevent of natural disasters ($S_t = 0$)? In other words, what happens to the environment when in equilibrium innovation occurs only in the green sector?

Given the results in Acemoglu et al. (2012) and using equations (8) and (9), one can conclude that when all scientists are allocated to the clean sector ($s_{ct} = 1$), the production of dirty inputs (Y_{dt}) grows at a rate $(1 + \gamma\eta_c)^{\alpha+\gamma} - 1 > 0$, if the the two inputs are weak substitutes (i.e. $1 < \epsilon < 1/(1-\alpha)$). In contrast, when inputs are strong substitutes ($\epsilon > 1/(1-\alpha)$), Y_{dt} behaves in the long run as $A_{ct}^{\alpha+\varphi}$, which in turns decreases over time.

The above results imply that appropriate climate policies are able to prevent environmental catastrophes only when dirty and clean inputs are strong substitutes. More precisely, C&C policies are always successful in avoiding S_t to reach zero, whereas M-B solutions are effective only if they are implemented at the right time. If inputs are weak substitutes instead, final good production requires increasing amount of dirty inputs which cannot be replaced by clean ones, even though the productivity of dirty machines keeps constant. As a results an environmental disasters looks inevitable.

Given such a gloomy perspective, is there an environmental policy that work even if clean and dirty inputs are weak substitutes? The next proposition provides a positive answer:

Proposition 4. *Let $\hat{\kappa}$ be a C&C policy imposing a fixed ceiling on the use of dirty inputs Y_{dt} such that*

$Y_{dt} \leq \hat{\kappa}$. *Then, there exists a finite* $\hat{\kappa}' > 0$ *such that* $\forall\, \hat{\kappa} \in (0, \hat{\kappa}')$ *the policy always redirects technical change towards the green sector and prevents an environmental disaster.*

Proof. See Appendix A.4 □

The claim follows straightforwardly from proposition 3 and equation (6).[8] A fixed-ceiling C&C policy is always effective, even when the dirty and clean inputs are complementary, whereas in Acemoglu et al. (2012) disasters can be avoided only by switching off economic growth.

The results obtained from our policy exercises are summarized in Table 2. Only a C&C policy fixing the maximal amount of dirty inputs in the economy is always able to guarantee both technical change redirection and avoidance of environmental disasters. Marked-based solutions are effective only within their limited window of opportunity but, in general, they fail to guarantee disasters' prevention.

The lack of symmetry between the effects produced by the two types of policy instruments is attributable to the role of path dependence in technical change. In the laissez-faire and M-B cases, innovation has a self-perpetuating nature grounded on its own past success. This makes more and more difficult over time the transition to a clean equilibrium. On the contrary, in the C&C scenario, the labour destroying effect of innovation (see equation 13) exactly compensates the direct productivity effect, thus removing the dependence of this policy from the relative productivity gap of machines.[9]

5 Conclusions

In this work, we have extended the model developed in Acemoglu et al. (2012) to compare the impact of environmental market-based (M-B) policies with command-and-control (C&C) ones. M-B

[8]The maximum Y_{dt} allowed by the regulation policy at time t must be below S_{t-1}/ξ in order to prevent the realization of an environmental disaster.

[9]Also the use of a carbon tax equal to the price of dirty inputs would completely remove expected profits in that sector, triggering a complete shift toward the clean technology. However, this scenario is not feasible from a political perspective (see also Goulder and Parry, 2008, on the generally low political workability of carbon taxes) and meaningless from an economic one (see Section 4.2).

Table 2: Policy intervention and guaranteed results

	redirection of technical change		prevention of environmental disaster	
	weak substitutes	strong substitutes	weak substitutes	strong substitutes
Market based				
Taxes	window only	window only	no	window only
Subsidies	window only	window only	no	window only
Taxes and Subsidies	window only	window only	no	window only
Optimal Policy	window only	window only	no	window only
Command and control				
Relative share	yes	yes	no	yes
Absolute limit	yes	yes	yes	yes

policies are grounded on a carbon tax and a subsidy to the green sector, whereas C&C interventions fix limits to the production of dirty inputs and related greenhouse gas emissions.

We find that market-based policies are not always successful to redirect technical change from the dirty to the green sector. Given the cumulativeness of technical change (Aghion et al., 2015), time is fundamental and there is a limited window of opportunity to trigger a green transition. Indeed, if the productivity gap between the dirty and green sectors becomes too high, M-B policies becomes ineffective. The time for an effective intervention gets shorter if the two inputs are "weak" substitutes. In the latter case, market-based policies are never able to prevent environmental catastrophes. On the contrary, if the dirty and green inputs are "strong" substitutes, *timely* M-B interventions can successfully avoid the occurrence of disasters.

Command-and-control policies can *always* redirect technical change toward the green sector. In that, they are more effective than market-based interventions. Such result occurs because M-B policies work only via the market size channel (larger input sector stimulates innovation), whereas C&C interventions also affect the relative price and are not limited by the productivity gap between dirty and green technologies (cf. Equation 7). This explains why the evolution of the technologies do not affect the success of command-and-control policies, which are also *always* effective in preventing environmental disasters when inputs are strong substitutes. If the dirty and green inputs are weak substitutes, the only environmental policy that always allows to avoid a climate catastrophe is a C&C intervention fixing an absolute limit on the use of polluting inputs.

Our findings support the current behavior of governments, which are timid in introducing carbon pricing, relying instead on regulations that redirect investment towards clean capital, such as stricter energy efficiency standards on new capital and buildings (IEA, 2016). Beyond inducing the development of clean technologies, such regulatory measures are effective in preventing the occurrence of environmental disasters.

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A Mathematical Appendix

A.1 Proposition 1

We divide the proof in two parts. First, we show that the policy scheme (q_{ct}, t_{dt}) with $q_{ct} \leq \vartheta < +\infty$ and $t_{dt} \leq \theta < p_{dt}$ is not able to redirect technical change, then we characterize the unique equilibrium.

In the model, the more the dirty sector is profitable, the more researchers devote effort to innovate therein, the more dirty machines become productive and the relative share of dirty inputs increases. Under the policy scheme (q_{ct}, t_{dt}), the profitability of the two sectors is determined by three elements, namely the productivity ratio, the price of dirty inputs and the carbon tax:

$$\begin{aligned} \frac{\Pi_{ct}}{\Pi_{dt}} &= (1+q_{ct})\frac{\eta_c}{\eta_d}\left(\frac{p_{dt}-t_d}{p_d}\right)^{-\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{-\varphi-1}\frac{A_{ct-1}}{A_{dt-1}} \\ &= (1+q_{ct})\frac{\eta_c}{\eta_d}\left(\frac{p_{dt}-t_d}{p_d}\right)^{-\epsilon}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{-\varphi-1}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}, \end{aligned} \tag{15}$$

where the second line follows from (5) and where $\varphi = (1-\epsilon)(1-\alpha) < 0$ and $s_{ct} = 1 - s_{dt}$. Let

$$f(s) = (1+q_{ct})\frac{\eta_c}{\eta_d}\left(\frac{p_{dt}-t_d}{p_d}\right)^{-\epsilon}\left(\frac{1+\gamma\eta_c s}{1+\gamma\eta_d(1-s)}\right)^{-\varphi-1}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi},$$

where $s = s_{ct} = 1 - s_{dt}$. If $f(0) < 1$, then $s = 0$ is an equilibrium where all scientists devote their effort toward the dirty sector.

By assumption the economy is initially stacked in the bad equilibrium where productivity-improving innovations take place only for dirty machines. A carbon tax on the production of dirty inputs t_{dt}, and a subsidy q_{ct}, introduced at time T, are able to redirect technical change if they guarantee $f(0) > 1$, which corresponds to:

$$(1+q_{cT})\frac{\eta_c}{\eta_d}\left(\frac{p_{dT}-t_{dT}}{p_d}\right)^{-\epsilon}(1+\gamma\eta_d)^{\varphi+1}\left(\frac{A_{cT-1}}{A_{dT-1}}\right)^{-\varphi} > 1. \tag{16}$$

We analyze the case where the government provides the maximum possible subsidy, $q_{cT} = \vartheta$. If the tax is not able to redirect technical change in such scenario, then it would not be effective for all $q_{cT} < \vartheta$ as well (results do not change if the maximum available tax is fixed and one studies how subsidy affects technical change). The tax is effective in redirecting technical change if the following condition is satisfied:

$$t_{dT} > p_{dT} - \left[(1+\vartheta)\frac{\eta_c}{\eta_d}(1+\eta_d\gamma)^{\varphi+1}\left(\frac{A_{cT-1}}{A_{dT-1}}\right)^{-\varphi}\right]^{\frac{1}{\epsilon}} p_{dT}. \tag{17}$$

Given the productivity of machines in the dirty sector r, define $g(r)$:

$$g(r) := p_{dT} - \left[(1+\vartheta)\frac{\eta_c}{\eta_d}(1+\eta_d\gamma)^{\varphi+1}\left(\frac{A_{cT-1}}{r}\right)^{-\varphi}\right]^{\frac{1}{\epsilon}} p_{dT}.$$

$g(r)$ is a continuous function in $(0;+\infty)$ and satisfies:

$$\lim_{r\to+\infty} g(r) = p_{dT}.$$

Without loss of generality, let $\theta = p_{dT} - \delta$ with $\delta \in \mathbf{R}^+$. Then, using the definition of limit, for all $\delta > 0$, it exists a $\bar{A}_d \ll \infty$, such that for all, $r > \bar{A}_d$, one obtains

$$p_{dT} - g(r) < \delta,$$

which in turns implies $g(r) > p_{dT} - \delta$. Finally, there exists a finite r and a sufficiently low δ such that, in order to redirect technical change, it is required

$$t_{dT} > g(r) > \theta,$$

which is impossible because it contradicts our assumptions.

Now let us show that the equilibrium where all researchers are employed in the dirty sector ($s = 0$) is also the unique equilibrium when A_{dT-1} is sufficiently large. Two cases must be distinguished.

First, if $1 + \varphi > 0$, then $f(s)$ is strictly decreasing in s and $f(0) < 1$ guarantees that $s = 0$ is the unique equilibrium. The previous condition can be rewritten as follows:

$$f(0) = (1+\vartheta)\frac{\eta_c}{\eta_d}\left(\frac{p_{dT} - t_{dT}}{p_{dT}}\right)^{-\epsilon}(1+\eta_d\gamma)^{\varphi+1}\left(\frac{A_{cT-1}}{A_{dT-1}}\right)^{-\varphi} < 1,$$

which implies

$$\left(\frac{A_{cT-1}}{A_{dT-1}}\right)^{-\varphi} < \frac{\eta_d}{\eta_c(1+\vartheta)}\left(\frac{p_{dT} - t_{dT}}{p_{dT}}\right)^{\epsilon}\left(\frac{1}{1+\eta_d\gamma}\right)^{\varphi+1} = \Psi, \tag{18}$$

where $\Psi > 0$. As $\epsilon > 1$, the left hand side of (18) is a continuous function monotonically decreasing in A_{dT-1} which tends to 0 as the productivity of machines in the dirty sector becomes larger and larger. This proves that for a sufficiently large A_{dT-1}, the unique equilibrium allocation of scientists satisfies $s = 0$.

Now consider the second case where $1 + \varphi < 0$. As $f(s)$ is strictly increasing in s, the unique equilibrium is $s = 0$ only if $f(0) < f(1) < 1$, where the first inequality is obviously satisfied. Consider the second inequality:

$$f(1) = (1+\vartheta)\frac{\eta_c}{\eta_d}\left(\frac{p_{dT} - t_{dT}}{p_{dT}}\right)^{-\epsilon}(1+\eta_c\gamma)^{-\varphi-1}\left(\frac{A_{cT-1}}{A_{dT-1}}\right)^{-\varphi} < 1.$$

Accounting for the time the tax is introduced and after some algebra it becomes

$$\left(\frac{A_{cT-1}}{A_{dT-1}}\right)^{-\varphi} < \frac{\eta_d}{\eta_c(1+\vartheta)}\left(\frac{p_{dT} - t_{dT}}{p_{dT}}\right)^{\epsilon}(1+\eta_c\gamma)^{\varphi+1} = \Psi',$$

with $\Psi' > 0$. Analogously to the previous case, it is easy to see that for a sufficiently high A_{dT-1} the equilibrium $s = 0$ is unique. Finally, if $1 + \varphi = 1$ then $f(s) \equiv f$ is constant and $f < 1$, surely verified for some large A_{dT-1}, is sufficient to obtain the unique equilibrium $s = 0$.

A.2 Proposition 2

The proof relies upon the nature of the stochastic process characterizing the evolution of machines' productivity, i.e.

$$A_{jt} = \begin{cases} (1+\gamma s_{jt})A_{jt-1}, & \text{with probability } \ \eta_j \\ A_{jt-1}, & \text{with probability } \ (1-\eta_j) \end{cases}$$

with $j = \{c, d\}$ as usual. As $E(A_{jt} \,|\, A_{j0}, ..., A_{jt-1}) \geq A_{jt-1}$ for all t, A_{jt} is a submartingale. This implies that the expected productivity of dirty machines explodes as time goes by if no emission control policy is undertaken. Moreover, let $\bar{A}_d$ be the lowest threshold such that any $A_{dT} > \bar{A}_d$ prevents inequality (16) to be satisfied while let (18) holding. In principle, $\bar{A}_d$ can take any real number. However, we assume, without loss of generality, that $\bar{A}_d$ is an element of the sequence $\{A_{d0}(1+\gamma)^n\}_{n=0}^{\infty}$. Since the economy starts from the initial bad equilibrium where $s_{d0} = 1$, the probability of exceeding the threshold can be written as

$$\begin{aligned} P(A_{dt} > \bar{A}_d, \,|\, t \in \mathbf{N},\, t > n) &= 1 - \sum_{i=0}^{n-1} \binom{t}{t-i} (1-\eta_d)^i \eta_d^{t-i}, \\ &= 1 - \left[q^t + tpq^{t-1} + ... + \frac{t(t-1)...(t-n+2)}{(n+1)!} p^{n-1} q^{t-n-1} \right], \end{aligned}$$

where $q = \eta_d$ and $p = 1 - \eta_d$. Now let us study the probability limit of exceeding the threshold as time goes by. Since $0 < \eta_d < 1$, one obtains that

$$\lim_{t \to +\infty} P(A_{dt} > \bar{A}_d \,|\, t \in \mathbf{N},\, t > n) = 1.$$

A.3 Proposition 3

First, we notice that equation (14) can be expressed as

$$g(s) = \frac{\eta_c}{\eta_d} \kappa^{\frac{\epsilon-1}{\epsilon}} \left(\frac{1+\eta_c s}{1+\eta_d(1-s)} \right)^{-1}, \tag{19}$$

where $s = s_{ct} = 1 - s_{dt}$. As the economy is stacked in the bad equilibrium, in order to redirect technical change, a command-and-control policy κ should satisfy $g(0) > 1$. This implies that $s = 1$ is the unique equilibrium allocation of scientists. Imposing previous condition ($g(0) > 1$) one obtains:

$$\frac{\eta_c}{\eta_d}\kappa^{\frac{\epsilon-1}{\epsilon}}(1+\eta_c)^{-1} > 1. \tag{20}$$

The latter condition can be easily expressed as

$$\begin{aligned} &\kappa > \bar{\kappa} \\ &\bar{\kappa} = \left(\tfrac{\eta_d(1+\eta_c)}{\eta_c}\right)^{\frac{\epsilon}{\epsilon-1}}. \end{aligned} \tag{21}$$

Notice that $\bar{\kappa}$ is strictly positive and does not depend on the relative productivity of machines (at any instant of time). In addition as $\kappa \in (0, \infty)$, there will always be a C&C policy $\kappa > \bar{\kappa}$ that successfully redirects technical change.

A.4 Proposition 4

Let us start noticing that Assumption 1 implies the command-and-control policy scheme to be binding, that is $Y_{dt} = \hat{\kappa}$. Therefore, by solving the model in section 4.3, one obtains that (similarly to proposition 3's proof) technical change is redirected towards the "green" equilibrium $s_{ct} = 1$ and $s_{dt} = 0$ if

$$\frac{Y_{ct}}{\hat{\kappa}} > \left(\frac{\eta_d(1+\eta_c)}{\eta_c}\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{22}$$

which easily translates in

$$\hat{\kappa} < Y_{ct}\frac{\eta_c}{\eta_d(1+\eta_c)}^{\frac{\epsilon}{\epsilon-1}}. \tag{23}$$

Since Y_{ct}, η_c, η_d and ϵ are strictly positive, there always exists an absolute C&C policy $\hat{\kappa} > 0$ able to redirect technical change.

An environmental disaster is avoided if S_t is permanently positive after policy intervention. Since Y_{dt} is bounded from above, it suffices that $\hat{\kappa} < S_{t-1}/\xi$ to prevent an environmental catastrophe. Therefore, any C&C policy such that

$$\begin{aligned} &\hat{\kappa} < \hat{\kappa}' \\ &\hat{\kappa}' = \min\left(S_{t-1}/\xi,\, Y_{ct}\tfrac{\eta_c}{\eta_d(1+\eta_c)}^{\frac{\epsilon}{\epsilon-1}}\right) \end{aligned} \tag{24}$$

always guarantees the redirection of technical change towards the clean sector and the avoidance of environmental disaster.